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A SELF-SIMILAR SOLUTION FOR FAN JETS WITH AN ARBITRARY DEGREE OF SWIRLING

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Solutions are known [1-4] to the problem of the propagation of swirled fan (radial) jets into a submerged space. Functions which are valid at a distance considerably exceeding the radius of the round slit, where the jet is always weakly swirled, are obtained in [1, 2]. In the search for a solution for a jet discharging from an infinitely narrow slit of finite radius the assumption of weak swirling of the jet was introduced in [3] as an auxiliary assumption. In [4], where several terms of an asymptotic expansion by inverse powers of the distance from the nozzle were found for a laminar jet with a considerable swirling, the question of the determination of the integration constants remains open.

In the present paper it is shown that the problem of the propagation of a fan jet discharging from an infinitely narrow slit of finite radius has a self-similar solution for any degree of swirling of the jet.

§1. In the approximation of boundary-layer theory the equations describing the flow in swirled laminar or turbulent fan jets of incompressible liquid have the following form in the cylindrical coordinate system x, y, φ (the x axis is directed perpendicular to the axis of symmetry and φ is the polar angle)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\frac{w^2}{x}=\frac{1}{\rho}\frac{\partial \tau_x}{\partial y}; \tag{1.1}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+\frac{uw}{x}=\frac{1}{\rho}\frac{\partial \tau_\varphi}{\partial y}; \tag{1.2}$$

$$\frac{\partial (xu)}{\partial x}+\frac{\partial (xv)}{\partial y}=0, \tag{1.3}$$

where u, v, and w are the components of the velocity vector in the directions of the x, y, and φ axes; τ_x and τ_φ are the components of the shear stress of friction in the directions of the x and φ axes; ρ is the density of the liquid.

First let us consider a free submerged jet. Then the system (1.1)-(1.3) must be integrated with the following boundary conditions:

$$\begin{cases} u=0, \quad w=0 & \text{at} \quad y=\pm\infty; \\ \dfrac{\partial u}{\partial y}=0 & \text{at} \quad y=0. \end{cases} \tag{1.4}$$

The goal of the present report is to find a self-similar solution, and therefore the initial condition loses its importance. The two integral conditions of conservation needed for complete determinacy of the problem will be obtained in the course of the solution.

We will adopt the widely prevalent hypothesis that the following relationship is valid not only for laminar flows but also for turbulent flows of the boundary-layer type:

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$$\frac{\tau_x}{\partial u/\partial y} = \frac{\tau_\varphi}{\partial w/\partial y}. \quad (1.5)$$

§2. Equations (1.1)-(1.3) and the boundary conditions (1.4) do not impose restrictions on the assumption of similarity of the profiles of the longitudinal velocity components in each cross section. Thus, we set

$$w(x, y) = \lambda(x)u(x, y). \quad (2.1)$$

Substituting (1.5) and (2.1) into (1.1) and (1.2), we ascertain that with

$$\lambda = (Dx^2 - 1)^{-1/2}, \quad (2.2)$$

where D is a constant, Eqs. (1.1) and (1.2) are reduced to the same form

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\lambda^2 u^2}{x} = \frac{1}{\rho}\frac{\partial \tau_x}{\partial y}. \quad (2.3)$$

Consequently, the second of these equations can be omitted from further consideration.

We perform the following transformation of variables of the system of equations (2.3) and (1.3):

$$\begin{cases} x_1 = \int\limits_{x_0}^{x} f(x/x_0)\,dx, \quad y_1 = y\omega x/x_0, \\ u_1 = u/\omega, \quad v_1 = \dfrac{x}{x_0 f(x/x_0)}\left[v + \dfrac{yu}{x\omega}\dfrac{d(x\omega)}{dx}\right], \end{cases} \quad (2.4)$$

where $\omega = \omega(x) = (1 + \lambda^2)^{-1/2}$; $f(x/x_0)$ is some function defined below; $x = x_0$ corresponds to the initial cross section, to the radius of the round slit from which the jet escapes, in particular. As a result of the application of the transformation (2.4) to Eqs. (2.3) and (1.3) we obtain the system

$$\begin{cases} u_1\dfrac{\partial u_1}{\partial x_1} + v_1\dfrac{\partial u_1}{\partial y_1} = \dfrac{x}{\rho\omega x_0 f(x/x_0)}\dfrac{\partial \tau_x}{\partial y_1}, \\ \dfrac{\partial u_1}{\partial x_1} + \dfrac{\partial v_1}{\partial y_1} = 0, \end{cases} \quad (2.5)$$

which exactly coincides with the system of equations describing the flow in plane jets if

$$\tau_1 = \frac{x\tau_x}{\omega x_0 f(x/x_0)}. \quad (2.6)$$

The boundary conditions (1.4) are transformed into the boundary conditions for a plane submerged jet.

Suppose that one is able to choose a function $f(x/x_0)$ such that (2.6) is satisfied. Then the problem of the propagation of a free swirled fan jet discharging from an infinitely narrow slit into a submerged space, the transformation (2.4) together with (1.5) and (2.1), is reduced to the problem of a plane free jet source. The latter is self-similar for both laminar [5] and turbulent [6] forms of motion. Consequently, the original problem also has a self-similar solution.

Without writing out the well-known [5, 6] solutions for plane jet sources, we only recall that these solutions are determined by a single quantity, apart from the physical characteristics of the liquid: the impulse of the jet. Let us express this quantity through the integral characteristics of a swirled fan jet.

The integral condition of conservation which is valid for a plane submerged jet is

$$\rho\int\limits_{-\infty}^{+\infty} u_1^2 dy_1 = K = \text{const},$$

which in the variables describing the propagation of a fan jet, in accordance with (2.4), has the form

$$2\pi\rho x\sqrt{\frac{1+\lambda^2}{1+\lambda_0^2}}\int\limits_{-\infty}^{+\infty} u^2 dy = J = \text{const}, \quad (2.7)$$

where

$$J = 2\pi\rho x_0 \int_{-\infty}^{+\infty} u^2(x_0, y)\, dy;$$

$\lambda = \lambda_0$ at $x = x_0$ while the quantities J and K are related by the equation

$$K = \frac{J\sqrt{1+\lambda_0^2}}{2\pi x_0}. \tag{2.8}$$

From the integration of (1.2) across the jet we immediately get

$$2\pi\rho x^2 \int_{-\infty}^{+\infty} uw\,dy = L = \text{const}$$

or, using (2.1) and (2.7),

$$\lambda x J\sqrt{\frac{1+\lambda_0^2}{1+\lambda^2}} = L. \tag{2.9}$$

Taking $\lambda = \lambda_0$ at $x = x_0$ in (2.9) and (2.2), we obtain

$$\lambda_0 = \frac{L}{Jx_0}, \quad D = \frac{1+(Jx_0/L)^2}{x_0^2}. \tag{2.10}$$

Thus, the qualtity K, which enters into the solution for a plane jet, is fully determined through the integral characteristics of a fan jet.

Let us find the form of the function $f(x/x_0)$ which is needed later. In the case of laminar jets (μ is the coefficient of dynamic viscosity)

$$\tau_1 = \mu\frac{\partial u_1}{\partial y_1} = \frac{\mu x_0}{\omega^2 x}\frac{\partial u}{\partial x} = \frac{x_0\tau_x}{\omega^2 x},$$

which in conjunction with (2.6) determines $f(x/x_0) = \omega(x/x_0)^2$.

In a discussion of free turbulent flows one can neglect molecular transfer in comparison with turbulent transfer. Using the generalized mixing-length hypothesis of [7], for example, we write

$$\tau_x = \rho l^2\sqrt{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2}\frac{\partial u}{\partial y}, \quad \tau_1 = \rho l_1^2\left(\frac{\partial u_1}{\partial y_1}\right)^2. \tag{2.11}$$

We assume that the mixing lengths l and l_1 are proportional to the corresponding characteristic transverse sizes δ and δ_1; i.e., $l = c_0\delta$ and $l_1 = c_1\delta_1$, where c_0 and c_1 are empirical constants. Comparing (2.6) and (2.11) and allowing for (2.1) and (2.4), for the turbulent form of motion we will have

$$f(x/x_0) = n^2 x/x_0. \tag{2.12}$$

Here the factor $n^2 = (c_0/c_1)^2$ allows for the possibility that the constants of proportionality between the mixing length and the characteristic transverse size for fan and plane jets may not coincide. The form (2.12) of the function $f(x/x_0)$ is retained when the most simple hypotheses of turbulence are used.

It also turns out to be possible to obtain a self-similar solution for laminar semibounded fan jets discharging from infinitely narrow slits of finite radius and having an arbitrary degree of swirling. In the statement of the problem for a semibounded jet, in comparison with that for a free jet, only the boundary conditions are changed, and they now take the form

$$\begin{cases} u = w = v = 0 & \text{at} \quad y = 0, \\ u = w = 0 & \text{at} \quad y = \infty. \end{cases}$$

One can ascertain that the transformation (2.4) together with (2.1) and (2.2) reduces this problem to the plane problem of a semibounded jet discharging from an infinitely narrow slit. The latter has a self-similar solution, obtained in [8]. In this case the integral quantity

$$N = \rho\int_0^\infty u_1^2\left(\int_0^{y_1} u_1\,dy_1\right)dy_1 = \text{const}$$

entering into the solution for the plane jet is expressed through the integral characteristics of the swirled fan jet in a way analogous to (2.8) and (2.10):

$$N = \frac{E}{2\pi x_0} \sqrt{1 + \lambda_0^2},\ \lambda_0 = \frac{M}{E x_0},$$

where (see [2])

$$E = 2\pi\rho x_0^2 \int_0^\infty u^2(x_0, y) \left[\int_0^y u(x_0, y)\, dy \right] dy;$$

$$M = 2\pi\rho x^3 \int_0^\infty uw \left(\int_0^y u dy \right) dy = \text{const}.$$

In conclusion, we note that all the solutions found earlier for free swirled fan jets are obtained as particular cases from the results of the present work.

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STABILITY OF POISEUILLE FLOW IN AN ELASTIC CHANNEL

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The stability of a laminar boundary layer at a surface of the membrane type has been analyzed in [1, 2], while the stability of Poiseuille flow between membranes are analyzed in [3, 4]. Walls with a linear relationship between the perturbation of the pressure and the normal deformation of the surface were taken as the channel boundaries in [5]. The stability of the profile V = sin y ($0 \leq y \leq$) was analyzed numerically in [6]. The stability of Poiseuille flow in a channel whose walls are elastic plates is studied in the present report. In contrast to [3, 5, 6], pulsations of the friction at the channel walls are taken into account along with pressure pulsations, just as in [4]. It is shown that a significant reorganization of the regions of instability occurs when they are allowed for. A region of instability is found which exists for any finite Reynolds number.

A stream whose velocity profile is $V \equiv V_x = 1 - y^2$ in a channel with walls $y = \pm 1$ is analyzed (Fig. 1). For the normal and tangential displacements of the upper plate we have [7]

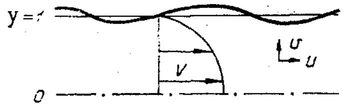

Fig. 1

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